

## **Numerical Exploration of Shear-Fracture-Related Rock Bursts Using a Strain-Softening Constitutive Law**

**By J. K. Whyatt and M. P. Board**

UNITED STATES DEPARTMENT OF THE INTERIOR



BUREAU OF MINES



**Mission:** As the Nation's principal conservation agency, the Department of the Interior has responsibility for most of our nationally-owned public lands and natural and cultural resources. This includes fostering wise use of our land and water resources, protecting our fish and wildlife, preserving the environmental and cultural values of our national parks and historical places, and providing for the enjoyment of life through outdoor recreation. The Department assesses our energy and mineral resources and works to assure that their development is in the best interests of all our people. The Department also promotes the goals of the Take Pride in America campaign by encouraging stewardship and citizen responsibility for the public lands and promoting citizen participation in their care. The Department also has a major responsibility for American Indian reservation communities and for people who live in Island Territories under U.S. Administration.

**Report of Investigations 9350**

# **Numerical Exploration of Shear-Fracture-Related Rock Bursts Using a Strain-Softening Constitutive Law**

**By J. K. Whyatt and M. P. Board**

**UNITED STATES DEPARTMENT OF THE INTERIOR**  
**Manuel Lujan, Jr., Secretary**

**BUREAU OF MINES**  
**T S Ary, Director**

**Library of Congress Cataloging in Publication Data:**

**Whyatt, J. K. (Jeffrey K.)**

Numerical exploration of shear-fracture-related rock bursts using a strain-softening constitutive law / by Jeffrey K. Whyatt and M. P. Board.

p. cm. — (Report of investigations; 9350)

Includes bibliographical references (p. 14).

Supt. of Docs. no.: I 28.23:9350.

1. Rock bursts—Mathematical models. 2. Shear (Mechanics)—Mathematical models. 3. Rock bursts—Coeur d'Alene Mountains Region (Idaho and Mont.) I. Board, M. P. (Mark P.) II. Title. III. Series: Report of investigations (United States. Bureau of Mines); 9350.

TN23.U43 [TN317] 622 s—dc20 [622'28] 90-15068 CIP

## CONTENTS

### Page

|   |    |
|---|----|
| Abstract .....  | 1  |
| Introduction .....  | 2  |
| Rock bursts and fracture zone formation in deep mines ..... | 2  |
| Conditions required for seismicity .....                    | 2  |
| Rock bursts and shear fracturing .....                      | 3  |
| Compressive failure of rock .....                           | 4  |
| Existing practice .....                                     | 5  |
| Strain-softening constitutive law .....                     | 5  |
| Numerical implementation of strain softening .....          | 7  |
| Simulation of laboratory tests .....                        | 8  |
| Simulation of mining .....                                  | 10 |
| Simulation of overhand mining .....                         | 11 |
| Simulation of underhand mining .....                        | 11 |
| Discussion .....  | 13 |
| Conclusions .....   | 13 |
| References .....  | 14 |

## ILLUSTRATIONS

|   |    |
|---|----|
| 1. Fracture types around deep underground stope .....   | 4  |
| 2. Characteristic failure and machine unloading curves .....  | 4  |
| 3. Average stress-compression curve for coal pillar .....   | 6  |
| 4. Uniaxial compression of slender cylinder with possible shear fractures .....   | 7  |
| 5. Effects of cohesion softening on numerically produced stress-strain response .....                                       | 8  |
| 6. Numerically produced shear failure in sample with 2:1 height-width ratio .....   | 9  |
| 7. Numerically produced shear failure in sample with 2:3 height-width ratio .....   | 9  |
| 8. Comparison of sample shape effects for uniaxial compression .....  | 9  |
| 9. Isometric view of overhand and underhand cut-and-fill mining .....   | 10 |
| 10. Portion of finite-difference grid showing previously mined section of ore body and first cut of overhand<br>stope ..... | 11 |
| 11. Progressive development of shear bands around overhand cut-and-fill stope .....   | 12 |
| 12. Progressive development of shear bands around underhand cut-and-fill stope .....  | 12 |

# **UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT**

ft foot

pct percent

in inch

psi pound per square inch

# NUMERICAL EXPLORATION OF SHEAR-FRACTURE-RELATED ROCK BURSTS USING A STRAIN-SOFTENING CONSTITUTIVE LAW

By J. K. Whyatt<sup>1</sup> and M. P. Board<sup>2</sup>

---

## ABSTRACT

Researchers at the U.S. Bureau of Mines have been exploring recent advances in the use of strain-softening constitutive laws that approximate shear fracturing. The purpose of these tests is to investigate shear-fracture-related rock bursts in hard-rock mines. The capabilities and remaining problems in the use of strain-softening laws in finite-element, distinct-element, and finite-difference methods are reviewed. Performance of the finite-difference implementation of strain softening is compared to classic experimental results and observations of rock mass behavior in the Coeur d'Alene Mining District of northern Idaho. The results show promise for developing methods to identify the general location and orientation of shear fractures and their sequence of occurrence, but such methods cannot be relied upon to quantify post-fracture loads or displacements accurately. Uncertainty over the amount of energy consumed by failing rock and the dependency of energy consumption on model mesh definition prevent direct application of the results to analyses of rock bursts.

---

<sup>1</sup>Mining engineer, Spokane Research Center, U.S. Bureau of Mines, Spokane, WA.

<sup>2</sup>Project engineer, Itasca Consulting Group, Inc., Minneapolis, MN.

## INTRODUCTION

The Coeur d'Alene Mining District of northern Idaho has a long history of mining, and several district mines have pursued nearly vertical veins of high-grade zinc, lead, and silver to depths of over a mile. Mining to ever-greater depths has been accompanied by an increase in the number of seismic events.<sup>3</sup> The seismic energy from some of these events has been sufficient to damage underground openings and surface structures. In this report, damaging seismic events caused by mining are referred to as rock bursts, whereas naturally occurring damaging seismic events are referred to as earthquakes.

Rock bursts pose major safety and productivity problems for deep mines in the Coeur d'Alene Mining District. With large bursts reaching a magnitude of 3 to 4 on the Richter scale, miner safety is being seriously threatened and entire sections of deep mines have been severely damaged. Despite the seriousness of the rock burst threat, considerable gaps remain in the fundamental knowledge of rock burst mechanisms. The U.S. Bureau of Mines has taken a lead role in investigating both rock burst mechanisms and engineering methods to reduce rock burst hazards. These investigations have included a historical

review of rock bursting in the Coeur d'Alene District (17)<sup>4</sup> and studies of rock burst mechanisms of major bursts at the Lucky Friday Mine, Mullen, ID (14), rock mass preconditioning techniques (15), and strategies for designing mines to avoid rock bursts (13).

This Bureau report examines a cohesion-softening constitutive law that produces strain-softening behavior, as well as recent advances in implementing strain softening in several numerical modeling programs, or solvers, in order to evaluate the ability of the programs to analyze shear-fracture rock bursting. The first section briefly discusses current theoretical thinking on rock burst mechanisms and reviews field observations of fracturing around deep mine openings. The second section explores the ability of a cohesion-softening constitutive law to follow the strain-softening behavior of rock undergoing shear fracture by simulating a classic laboratory experiment on sample shape effects on rock strength. The final section applies the cohesion-softening model to an analysis of the relative shear-fracture rock burst potential for underhand longwall and overhand cut-and-fill stopes typical of the Coeur d'Alene Mining District.

## ROCK BURSTS AND FRACTURE ZONE FORMATION IN DEEP MINES

The high in situ stresses encountered at depth and stress concentrations developed by the excavation of mine openings can combine to exceed rock mass strength, especially in the immediate skin of an opening. The result is inelastic rock mass deformation caused by shear fracturing, tensile fracturing, and/or slip along discontinuities in a zone surrounding a deep mine opening. The development of this fracture zone is generally not a problem, and fractured rock walls are a common sight in deep mines. Rock bolts, timber, and other ground control measures can be introduced to prevent falls of fractured ground and limit the extent of the fracture zone. However, rock failure may accelerate from a quasistatic process to a dynamic process that releases a shock (seismic) wave.

In this section, the conditions necessary to accelerate inelastic rock mass deformation and generate seismic events (rock bursts) are reviewed, after which the specific role of shear fracturing in rock bursting is examined.

### CONDITIONS REQUIRED FOR SEISMICITY

Salamon (23, p. 25) listed the preexisting conditions necessary to initiate a seismic event.

- I. A region in the rock mass must be on the brink of unstable equilibrium either because:
  - a) The presence of an appropriately loaded pre-existing geological weakness such as a joint, fault, dike or bedding plane; or because
  - b) The changing stresses are driving a volume of rock towards sudden failure; or because
  - c) Some support system, for example a system of pillars, approaches a state in which its unstable collapse is imminent.
- II. Some induced stresses must affect the region in question, and the magnitude of these stress changes,

<sup>3</sup>A seismic event is defined here as a sudden episode of inelastic rock deformation (a shear fracture, tensile fracture, or slip along a discontinuity) that produces elastic shocks (seismic waves).

<sup>4</sup>Italic numbers in parentheses refer to items in the list of references at the end of this report.



however small, must be sufficiently large to trigger off the instability.

III. Sudden stress change of sizeable amplitude must take place at the locus of instability to initiate the propagation of seismic waves.

IV. Substantial amounts of energy must be stored in the rock around the instability to provide the source of kinetic or seismic energy. The origin of this stored strain energy is work done by:

- a) Gravitational forces and/or
- b) Tectonic forces and/or
- c) Stresses induced by mining.

Early efforts to analyze rock bursts (25) concentrated on condition IV, the energy requirement. Calculation of the energy release rate, which reflects the increase in strain energy stored around an opening resulting from additional excavation, proved to be a valuable indicator of rock burst potential. This method received widespread attention in South Africa, and an empirical relationship between the energy release rate and the statistical expectation of rock burst severity was found.

More recently, condition III was used as the basis for developing the excess shear stress method (22). In this method, the safety factors for slip at various locations and orientations are determined. A low-safety factor suggests that a large drop in stress is possible during slip along a discontinuity or shear fracture. Both of these methods are applicable to large-scale mine planning.

Condition II is included as a means for differentiating between natural, tectonically induced seismic events and those induced by mining. This is an important point when developing a strategy for dealing with rock bursts. If mining activity is causing seismicity, then altering that activity should alter the occurrence and characteristics of the seismicity, including the number and size of damaging rock bursts. However, to the extent that seismicity is a naturally occurring product of tectonic processes in the Earth's crust, the occurrence of earthquakes that can threaten mining activity is largely independent of mining techniques. Thus, attention should be directed towards enhancing mine survivability rather than altering mining practice to avoid triggering rock bursts.

From a rock mechanics standpoint, condition I, the actual failure process, is the most demanding, but this process is still poorly understood. All failure mechanisms will involve some combination of shear fracturing, tensile fracturing, and slip along existing discontinuities. However, knowledge of these mechanisms, with the possible

exception of tensile fracturing, is primitive, especially when these mechanisms are part of an unstable failure process.

## ROCK BURSTS AND SHEAR FRACTURING

This study focused on shear fractures that satisfied Salamon's condition I. Although experience has shown this to be the hardest condition to analyze, there also appeared to be some major advantages to examining it. First, as Hedley (9) points out, different rock burst mechanisms vary in their seismic efficiencies, that is, their ability to convert stored strain energy to seismic energy. Thus, knowing that a potential rock burst has a shear-fracture mechanism can improve estimates of probable damage. Second, the suitability of rock burst countermeasures is sensitive to the type of rock burst mechanism. Countermeasures, such as changing stope geometry, adding stabilizing pillars, destress blasting, and injecting fluids under high pressure into faults, are not equally effective for both fault slip and shear fracture mechanisms.

Shear fractures are commonly observed around deep mine openings. High in situ and induced stresses fracture rock immediately surrounding the opening, thus shifting excess stress deeper into the rock mass. During sinking of the Caladay Shaft in the Coeur d'Alene Mining District of northern Idaho, Whyatt and Beus (27) monitored shaft closure driven by the development of a zone of fractured rock. Dilatancy from fracturing was observed to depend directly on excavation, but also showed a time-dependent property commonly referred to as static fatigue in laboratory tests. Miners reported numerous seismic events, ranging from barely audible rock noise to loud thumps, during the monitoring period.

A typical fracture zone (fig. 1) was mapped by Adams, Jaeger, and Roering (1), who classified three types of fracture: tension fractures (I) that may have secondary vertical slips, shear fractures (II), and secondary tensile fractures (III). These types were numbered in their presumed order of development. Adams, Jaeger, and Roering did not detect seismic activity during the development of this fracture zone.

Ortlepp (19) investigated a large isolated pillar where 15 rock bursts were recorded over a 4-year period. Magnitudes ranged from 1.7 to 3.4 on the Richter scale. Further excavation uncovered traces of more than 20 shear fractures. Ortlepp established that a direct relationship, in space and time, existed between excavation and the formation of shear fractures. The question of whether the bursts were the result of fracture formation or of subsequent slip along the fracture planes was harder to resolve. Ortlepp observed that the potential stress drop, a major factor in determining seismic energy release, was greater

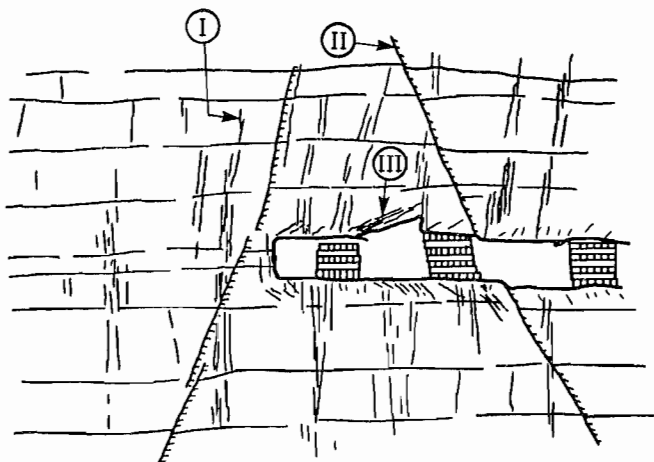


Figure 1.—Fracture types around deep underground stope. Tension fractures (I), shear fractures (II), and secondary tensile fractures (III). Adapted from reference 1.

when intact rock fractured than when rock slipped along a fracture. Thus he argued that bursts related to the formation or extension of shear fractures did occur, although slip bursts may also have taken place. The apparently random occurrence of these rock bursts in time were attributed to the time dependence of fracture zone development and local complexities in geologic structure.

More recently, Spottiswoode (24) measured the first motions of seismic events occurring in similar conditions and found that shear movement was indicated. The location, orientation, and direction of slip of many of these events were consistent with the type II fractures shown in figure 1. Other events showed slip, but in directions strongly influenced by dikes.

The hypothesis that shear fractures are associated with rock bursts as advanced by Ortlepp and Spottiswoode should not be seen as contradicting the lack of seismicity observed in fracture zone formation by Adams, Jaeger, and Roering. Rather, these two cases should be seen as opposite ends of a spectrum of possible seismic behavior. At one extreme, shear fracturing is an essentially quasi-static process, while at the other extreme, shear fracturing is sufficiently dynamic to generate a large shock wave. Shear fracturing is not the only mechanism capable of

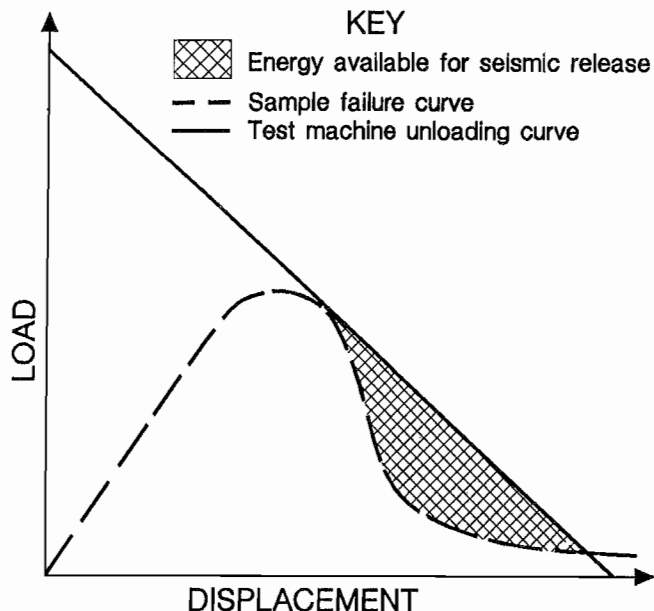


Figure 2.—Characteristic failure and machine unloading curves showing energy available for seismic release in a laboratory test.

generating shock waves in deep mines, but this report will be directed to a discussion of the necessary conditions for shear fracturing.

The violent shear failure of laboratory specimens in soft-testing machines is the most accessible example of a shear-fracture-based seismic event. To generate violent shear failure, a loading frame (or roof and floor strata) must converge with more energy than is consumed in the failing rock sample (or pillar). The difference between energy supply and consumption (fig. 2) is then available as kinetic energy. These energies and their differences are easily computed from frame unloading and rock failure curves.

Thus, the potential for this class of seismic event can be readily analyzed if the appropriate unloading and rock failure curves are available. Of these, the unloading curve is easiest to derive, especially when behavior of a loading frame (or roof and floor strata) is largely elastic. Determining the rock failure curve is much more difficult, especially when loading rate and load cycling effects on the rock failure curve are considered.

## COMPRESSIVE FAILURE OF ROCK

The large-scale failure of intact rock in a generally compressive stress field is dominated by the development of shear fractures. Physically, a shear fracture is only the end product of a complex deformation process in brittle

materials. Heterogeneous, brittle materials such as rock develop very complex stress fields at the granular scale as differences in modulus and strength cause loads to be redistributed. Although these local stress fields can reach

sufficient levels to generate microcracks either in individual grains or along grain boundaries, load levels may be significantly lower than those required for sample failure. These microcracks are observable on the specimen scale only as small increments of inelastic deformation.

As the shear strength of the rock is reached, the microcracking process localizes in a band that becomes a shear fracture (2). Experimental evidence suggests that the development of a through-going shear fracture actually lags attainment of peak load by a small amount. The dominance of shear fracturing in rock behavior is reflected in the pervasiveness of the Mohr-Coulomb envelope, a simple shear fracture failure criterion, in the rock and soil mechanics fields. However, this failure criterion defines only a threshold beyond which shear fracturing is active. The addition of a strain-softening constitutive law to the Mohr-Coulomb failure criterion enables the model to follow the reduction in load-carrying capacity of the rock by reducing cohesion as a function of plastic strain. As simple as this concept sounds, strain softening introduces subtle, but important, features of rock behavior and is very difficult to implement in numerical methods.

In this report, the complex intergranular and intragranular microcracking processes of shear fracture will be ignored. Instead, the focus will be on the ability of the strain-softening constitutive law to simulate some of the large-scale characteristics of a developing shear fracture.

## EXISTING PRACTICE

Engineers have used a variety of methods to approximate the extent of a fractured zone and the load-bearing capacity of a partially fractured portion of a rock mass. The most common approach has been to use the perfectly plastic associated-flow model developed for metals. However, this method has been shown to understate the extent of fracturing and, in turn, overestimate postpeak load-bearing capacity, especially for rock confined at low pressures. To compensate, rock strength parameters are often reduced on the basis of observations and/or measurements of the extent of the fracture zone as determined under similar conditions (21).

Wilson (28) recognized that strain softening was an important element in estimating the peak load-carrying capacity of coal mine pillars. The fractured skin of the pillar provides confinement to the inner intact portion, increasing the strength of the pillar core. Wagner (26) measured the response of a pillar to progressive loading and estimated loads in the core and perimeter sections of the pillar (fig. 3). Although this approach does not allow for exact computation of load-carrying capacity, it provides an excellent framework for understanding and predicting

pillar behavior and can be used to approximate the unloading curve of a failing pillar.

The presence of geologic features and discontinuities that may steepen the postpeak curve of a particular pillar are frequently hard to detect, but these hidden flaws are often associated with rock bursting. Thus, it is difficult to anticipate the rock burst potential of a particular pillar.

## STRAIN-SOFTENING CONSTITUTIVE LAW

A strain-softening constitutive law would simulate rock behavior beyond the failure criterion into the fracturing process. That is, the law would allow researchers to model the effects of shear fracturing and subsequent movement along fracture surfaces. A common form of a strain-softening constitutive law assumes that the rock behaves as a perfectly plastic Mohr-Coulomb material, except that cohesion is reduced as a function of plastic strain. In this way, the damage associated with postyield deformation is reflected in reduced cohesion until a residual value is reached. Similar forms of a strain-softening constitutive law could be envisioned for alternate failure envelopes, but such laws will not be considered in this report.

The cohesion-softening method is similar to Wilson's approach (28), except that it provides for a path between fully intact and fully fractured rock. In providing a path, the cohesion-softening method also introduces a positive feedback characteristic that reinforces and localizes plastic flow. That is, any part of a body that flows plastically will have its strength reduced, which encourages further plastic flow.

Difficulties are introduced by characteristic rock behaviors that are not represented in the strain-softening law. Chief among these difficulties is the lack of information about energy consumption during shear fracturing, the characteristic width of a shear fracture, the influence of small defects on fracture formation, and size effects.

In the laboratory compression test example, energy consumption is determined by the strain-softening parameters. That is, the plastic work required to destroy the cohesion in an element is analogous to the energy required for propagation of a shear fracture. Since general energy consumption guidelines are unavailable, the rate of cohesion reduction is calibrated to give a model test stress-strain curve that matches a measured curve. Any change in model conditions from the reference test that affects the energy consumed during shear fracturing will degrade model accuracy.

A geometrical factor related to energy consumption is the width of a shear fracture zone. The greater volume of damaged rock that exists in a wider zone will correspond to higher energy consumption, although energy

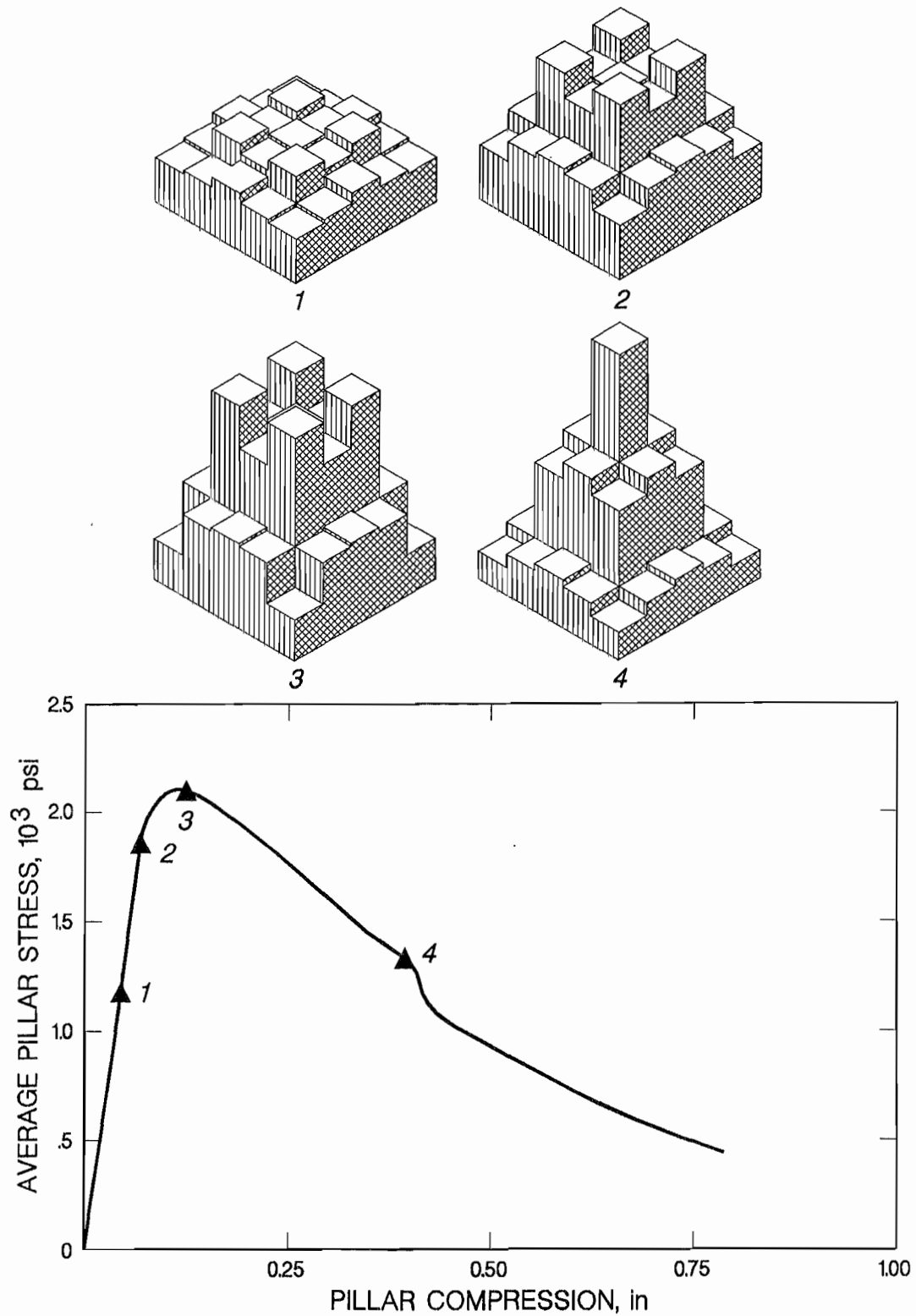


Figure 3.—Average stress-compression curve for coal pillar. Isometric histograms show vertical stress distribution at various stages (1-4) of pillar compression curves (after 26).

consumption will also be related to the offset distance of slip. The variety of shear fracture thicknesses observed in nature, from hairline cracks to region-wide fault zones, suggests that fracture width may be a major factor.

The absence of small defects in the assumed idealized material is important because defects channel fracturing among equally possible fractures. For example, consider the possible locations and orientations (strike) of a shear fracture during a uniaxial compression test (fig. 4). Any number of fractures could be activated simultaneously in an ideal material. This point in the load history, where several alternative fractures or deformation patterns are possible, is called a bifurcation point. In real rock, minute flaws will cause one particular fracture to propagate first.

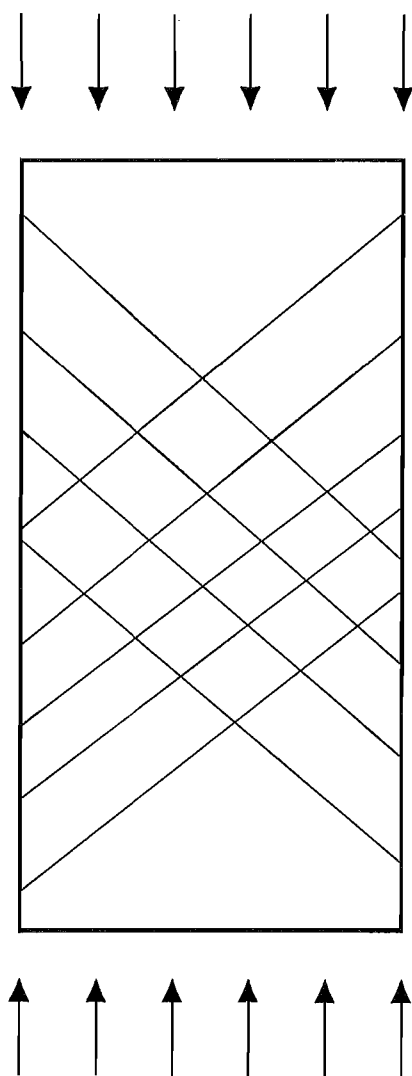


Figure 4.—Uniaxial compression of slender cylinder with possible shear fractures.

Finally, there is ample experimental evidence that the size of a sample affects its strength. Generally, large samples are found to be weaker than small samples up to a certain sample size, depending on rock type. Most researchers attribute this effect to flaws, with large samples being more likely to contain a "fatal flaw" than small samples. However, a comprehensive theoretical explanation of this phenomenon is still lacking.

Thus, there remain considerable fundamental problems in the application of a strain-softening constitutive law. In fact, one important result of writing a strain-softening law is that it highlights significant topics for continued experimental and theoretical research.

Despite these shortcomings, the strain-softening constitutive law is far from useless.

### NUMERICAL IMPLEMENTATION OF STRAIN SOFTENING

Investigators have attempted to incorporate plastic yield by using the strain-softening constitutive law in a variety of numerical methods, especially for real-world problems in mechanical, civil, and mining engineering. Despite the shortcomings discussed in the previous section, significant progress has been made. Presently, use of strain-softening concepts in discrete-element, finite-element, and finite-difference stress analyses have been at least partially successful.

The discontinuum distinct-element approach discretizes a body having joints that approximate grain boundaries. These joints are given sufficient initial strength to reflect the strength of the rock. Material between joints is allowed to deform elastically, but is considered to be infinitely strong, forcing all failure to occur as fractures along the joints. When these joints fail, their cohesion is reduced with a strain-softening scheme.

As a model specimen reaches its elastic limit, some preferentially oriented joints will slip locally and, with increasing load, join to form a shear failure. Applied on a very small scale, this is a model of the granular scale mechanics of elastic deformation and failure of rock. Practical computer run times require that model grains be magnified by several orders of magnitude, resulting in some loss of accuracy (16).

The continuum finite-element and finite-difference approaches rely on discretization of a solid into individual elements. This discretization introduces dependence on element size to achieve material behavior in strain-softening models. Because strain softening has been formulated as a reduction in strength with plastic strain, energy consumption and plastic displacement of a failing

element depend on element size. Thus, the rock failure curve is controlled by element size. This effect can be eliminated by scaling the cohesion loss: plastic strain ratio to changes in element size. The reference cohesion loss rate can be set such that the numerical failure curve matches a measured failure curve. Also, these continuum formulations of strain softening rely simply on plastic strain with no regard for direction. Thus, any element shape (except circular) will vary in softening behavior by direction. While this is not a serious problem for a square element, it quickly becomes a concern as the element-aspect ratio increases.

DeBorst (7-8) recently implemented strain softening in a finite-element program. The difficulties introduced by strain softening arise in the finite-element method as a singular (unsolvable) solution matrix at the bifurcation point. DeBorst demonstrated that the eigenvalues and eigenvectors of such a solution matrix indicate that alternate deformation patterns are available. He then selected the deformation pattern with the least potential energy for the numerical procedure to follow. Where identical energy release paths are available, an arbitrary decision must be made.

The finite-difference approach of the Fast Lagrangian Analysis of Continua (FLAC) program (5-6, 12) differs from the finite-element method in that it is a time marching rather than a matrix-solution method. The finite-difference nodes are analyzed singly and accelerated through a small time-step in response to loads exerted by adjacent elements. This is, in effect, a dynamic method with stress waves traveling through the model from node to node. Damping is added to move the system rapidly to equilibrium. Hence the solution is static, not dynamic.

The sequence in which nodes are examined and the presence of stress waves in the solution are sufficient to establish a unique path through the bifurcation point. In practice, these waves serve a role analogous to the small flaws present in real materials. As long as the magnitude of vibration is kept to reasonably low levels, the method finds the deformation pattern with the least potential energy. However, paths of equal potential energy can coexist in some cases, creating extra shear fractures.

The finite-difference approach was chosen for this investigation because it was available in a supported personal computer (PC) code that could be executed swiftly. The finite-element approach was not available in a supported code. The distinct-element approach was tried briefly, but was abandoned because of uncertainties associated with granular scale and the need for longer computer run times. However, both of these alternatives deserve further attention.

## SIMULATION OF LABORATORY TESTS

The strain-softening FLAC model was tested first by modeling a 2:1 (height by width) uniaxial compression test with lateral constraint at the platens. Although the constitutive law allows a nonassociated Mohr-Coulomb plasticity model with cohesion, friction, and dilatancy angles controllable as functions of plastic strain, a simplified case with associated plasticity and linearly varying cohesion was chosen. The rate of cohesion softening with plastic strain was varied (fig. 5A) to produce a range of complete stress-strain curves (fig. 5B). The peak strength was controlled by the cohesion defined for 0 pct plastic strain, while the brittleness of the postpeak response and the residual strength were functions of the rate of cohesion decay with plastic strain. In this way, behavior ranging from elastic-brittle to elastoplastic could be generated by proper adjustment of the cohesion-softening curve. The cohesion softening localized along the expected shear fracture orientations (fig. 6A), breaking the sample into four distinct wedges with two opposing shear planes (fig. 6B).

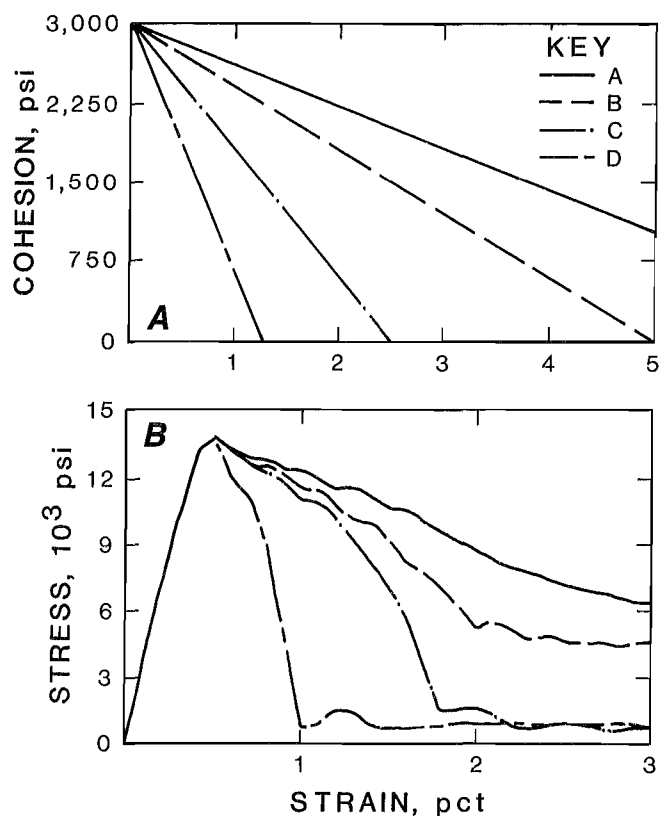


Figure 5.—Effects of cohesion softening on numerically produced stress-strain response. A, Assumed cohesion softening as function of plastic strain (that is, strain past yield); B, resulting stress-strain curves for sample with 2:1 height-width ratio in uniaxial compression. Note that brittleness of response is controlled by rate of decay of cohesion.

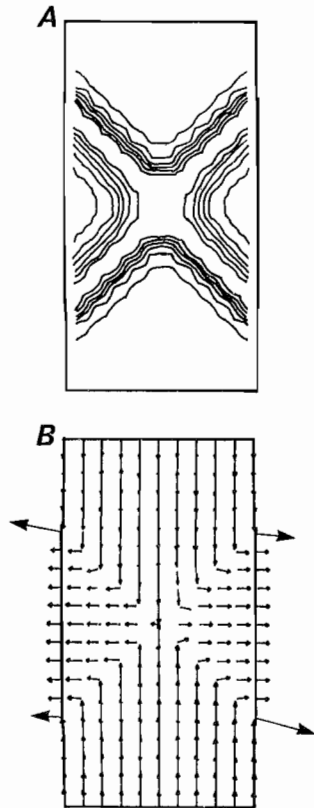


Figure 6.—Numerically produced shear failure in sample with 2:1 height-width ratio. A, Shear band formation is reflected by zones of reduced cohesion; B, velocity vectors show resulting wedges

The presence of two shear planes resulted from their numerical noninterference. In reality, flaws would weaken, and hence select, one of these fracture planes.

The capability of the model to follow actual rock behavior was tested further by examining the influence of sample shape. Decreasing the height-to-width ratio from 2:1 to 2:3, with constant element sizes, changed the pattern of shear fracturing considerably. An initial set of shear bands developed near the surface of the sample and stabilized. This set was followed by a second set of deeper shear bands (fig. 7A) that formed two extruded wedges (fig. 7B). Numerical tests were conducted for several sample shapes. The results (fig. 8B) compared favorably with the sample shape effects reported by Hudson, Brown, and Fairhurst (10) based on laboratory tests (fig. 8A). The two-dimensional model assumed an infinitely long prism, while the laboratory tests were conducted on cylinders.

A similar study (11) confirmed the ability of the model to simulate field measurements of coal pillar behavior and to produce results in agreement with Wilson's hypothesis (28). Thus, there is good reason to believe that the model, despite its shortcomings, captures an important aspect of rock behavior. Moreover, this behavior has generally been lacking from numerical models of rock mechanics.

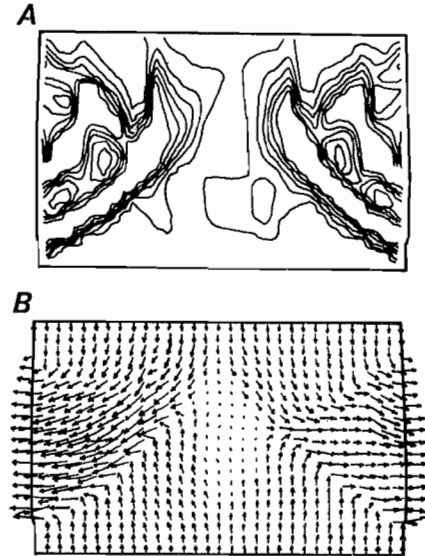


Figure 7.—Numerically produced shear failure in sample with 2:3 height-width ratio. A, Zones of reduced cohesion are localized in shear bands; B, wedges are clearly indicated by velocity vectors.

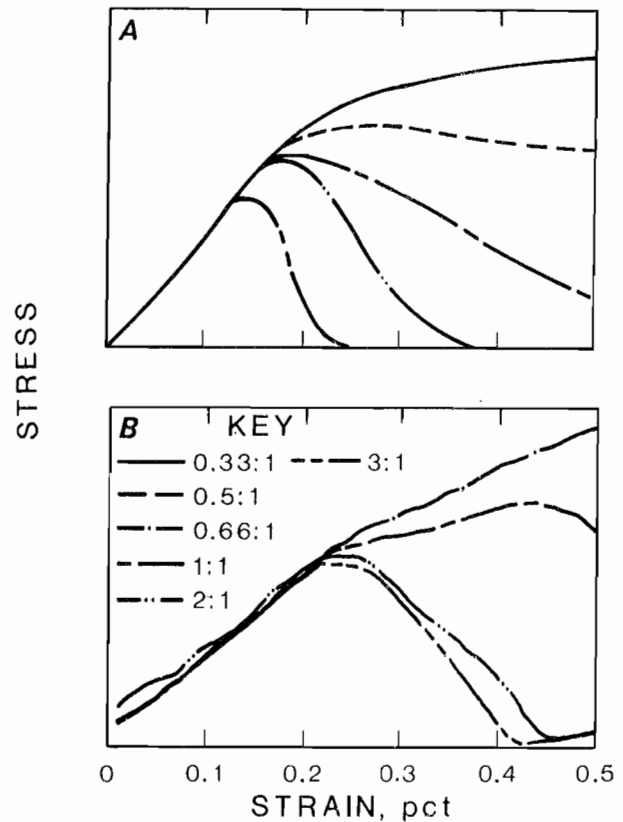


Figure 8.—Comparison of sample shape effects for uniaxial compression illustrate general correspondence of shape response of numerical model to laboratory tests. A, Experimental shape effects for laboratory uniaxial compression tests on marble (10); B, numerical shape effects from FLAC models of uniaxial compression tests.



## SIMULATION OF MINING

Results from the successful computer simulation of laboratory tests on hard rock and studies of the field behavior of a coal pillar suggest that the method may also be suitable for simulating fractures around deep mine openings. Unfortunately, reliable field measurements were not available for calibrating the softening and fracturing of rock in situ.

There were three other complicating factors. First, construction of a reasonable mesh for a stope section required a high degree of mesh gradation. However, it was impractical to assign a strain-softening rate to each element on the basis of its size, although this could be accomplished with software modifications. Second, the mesh required a departure from the square elements used in simulated laboratory tests, but there was no capability to induce strain softening at different rates in different directions within each element. Finally, the plastic strain that occurred in a single time-step was a much greater proportion of the plastic strain required for cohesion loss for these large elements than for the smaller elements used in the laboratory tests. This influenced convergence of the system and formation of shear bands. Because of these factors, mining simulations were undertaken using the same plastic strain-to-cohesion loss as in the simulations of the laboratory samples. Although this defeats the quantitative accuracy of the mining simulation, the exercise provided qualitative insight into the creation of shear fractures around deep stopes.

A stope in Hecla Mining Co.'s Lucky Friday Mine in the Coeur d'Alene Mining District near Mullan, ID, was chosen for the mining simulation. This mine is a current producer of lead-silver-zinc ore from a near-vertical vein at depths more than 5,000 ft below the ground surface. The rock mass consists of massive, brittle quartzite interbedded with soft argillites ranging from 1/2 to 4 in thick. Overhand cut-and-fill stoping has been the predominant mining method. Main levels are driven every 200 ft vertically, and the ore body is stoped vertically in several 10-ft-wide by 10-ft-high horizontal slices (fig. 9A). When each slice is completed, it is backfilled with uncemented mill tailings. Mining proceeds upward between levels, creating a sill pillar between the active stopes and the overlying mined-out stopes. Rock bursts may occur at any time, but are most likely to take place during extraction of the sill pillar. When the sill is reduced below about 100 ft in height, rock bursts of greater than 1 on the Richter scale may occur. While the largest rock bursts are thought to be caused by slip along bedding planes or preexisting faults, a significant number of smaller, but still dangerous, rock bursts appear to result from shear fracturing of intact rock.

In an attempt to reduce sill pillar bursts, Hecla changed to an underhand longwall cut-and-fill method of mining, which involves mining downward on a single face with each slice taken beneath the sandfill of the previous cut (fig. 9B). Therefore, no sill pillars are created. This change has been shown to reduce the peak energy release rate of mining (3).

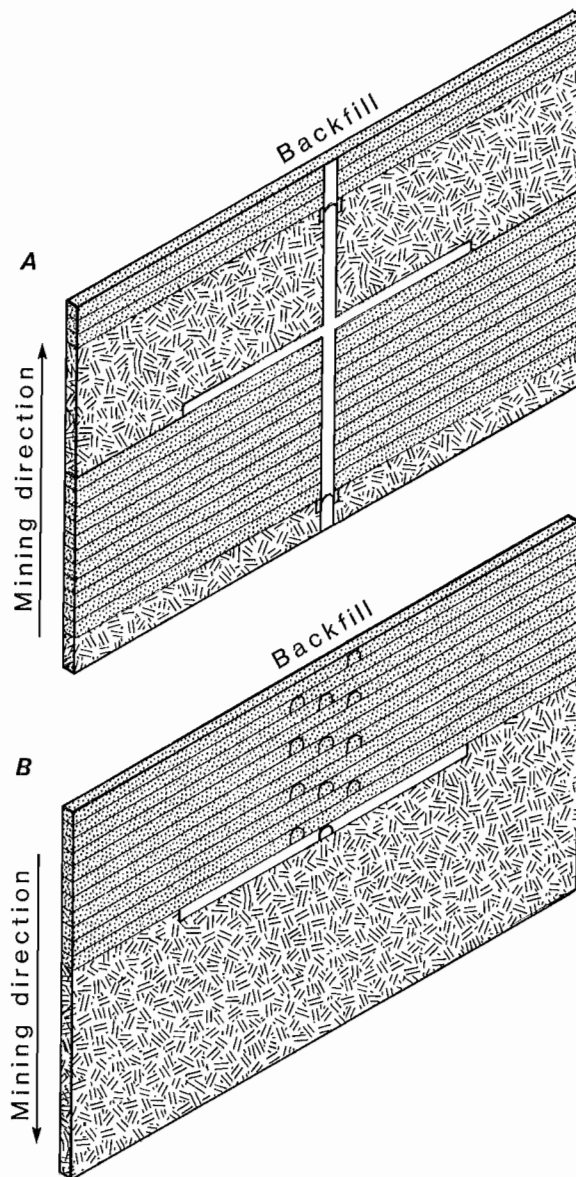


Figure 9.—Isometric view of overhand and underhand cut-and-fill mining. A, Overhand cut-and-fill mining where stopes are raised every 200 ft along ore body using lateral crosscut system; B, underhand cut-and-fill mining where stopes are accessed by ramp system.



Using the FLAC strain-softening constitutive model to identify the degree to which conditions for shear fracturing were affected, underhand and overhand mining methods were compared. In each case, the analysis began with a great length of the ore body removed (fig. 10) to approximate conditions in the upper levels of the mine. Sandfill was not incorporated in the model. The model was loaded with a 2:1 ratio of horizontal-to-vertical stress representing the existing underground stress field and brought to equilibrium prior to stope excavation. Because a two-dimensional model was used, a longwall face was assumed in both cases, and no analysis was made of stope advance along the vein or of staggered stope backs. Model rock properties were based on the complete stress-strain curve for quartzite as reported by Crouch (4). The cohesion was softened according to curve C in figure 5 to produce the desired response.

### SIMULATION OF OVERHAND MINING

The overhand simulation was performed by extracting one element at a time (that is, 20 ft, which was the equivalent of two stope cuts) and progressing updip. Figure 11 shows a series of contour plots of the plastic strain (strain past yield) produced as mining in the stope approached the previously mined level. Initially, the yield was confined primarily to the bottom of the upper, mined-out stope. The pillar began yielding as its height was

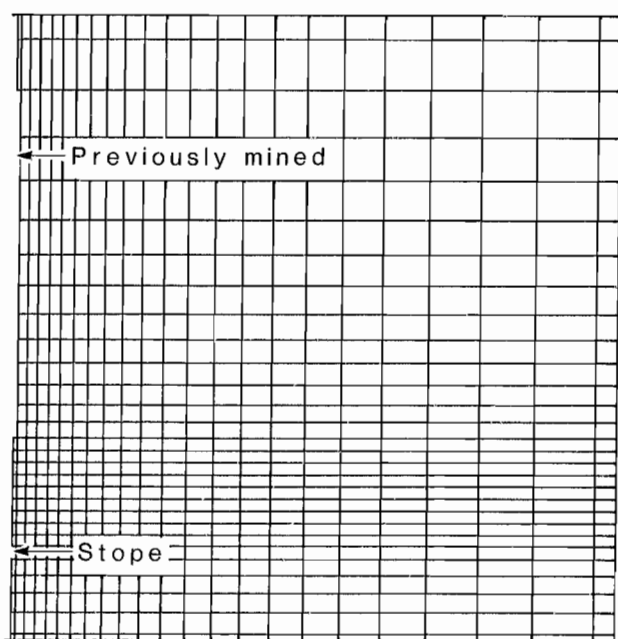


Figure 10.—Portion of finite-difference grid showing previously mined section of ore body and first cut of overhand stope.

reduced below 60 ft, and a second shear band at the bottom of the active stope began developing. The reduction of pillar height to 20 ft resulted in a dramatic failure, unloading of the pillar, and propagation of the bottom shear band 400 ft into the wall rock.

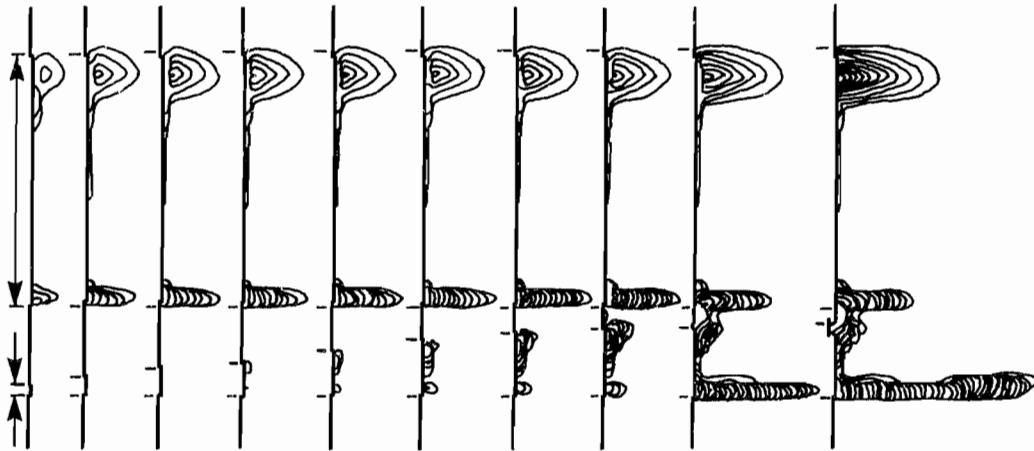
The model showed several similarities to the field behavior described previously. First, model pillar stresses peaked as sill pillar height was reduced to 40 ft. Further mining caused failure of the sill pillar, indicated by the formation of extensive shear bands. The sudden reaction of the model to this small excavation step appeared to correspond to rock bursting by shear rupture. This failure corresponded with the experiences of other mines in the district, indicating that seismic activity and rock bursts peaked when several cuts were left in the sill pillar (18). Thus, the final stope cuts were mined through highly fractured, and hence destressed, rock.

Second, as the pillar began to fail and unload, the lower shear zone propagated dramatically. This stage corresponded to field observations where rock bursts increased in number and magnitude at stope bottoms as fractured sill pillars failed, transferring load to intact rock below.

### SIMULATION OF UNDERHAND MINING

Simulation of underhand mining was begun with the same initial mining sequence as for the overhand simulation. Mining was then carried downward through the same block of ore. Figure 12 is a contour plot of plastic strain that shows the progressive development of a series of shear bands at the advancing face, as well as an outline of the yield zone at each mining step. As cuts were excavated, corresponding shear bands propagated about 250 ft perpendicular to the face advance. This periodicity of shear bands was largely a function of excavation, which was one element (20 ft or two cuts) at a time, and should not be taken as representative of fracture spacing in the field. Further model refinements are required before actual spacing can be defined. However, propagation of shear bands perpendicular to face advance was similar to the situation observed during South African field experience (20).

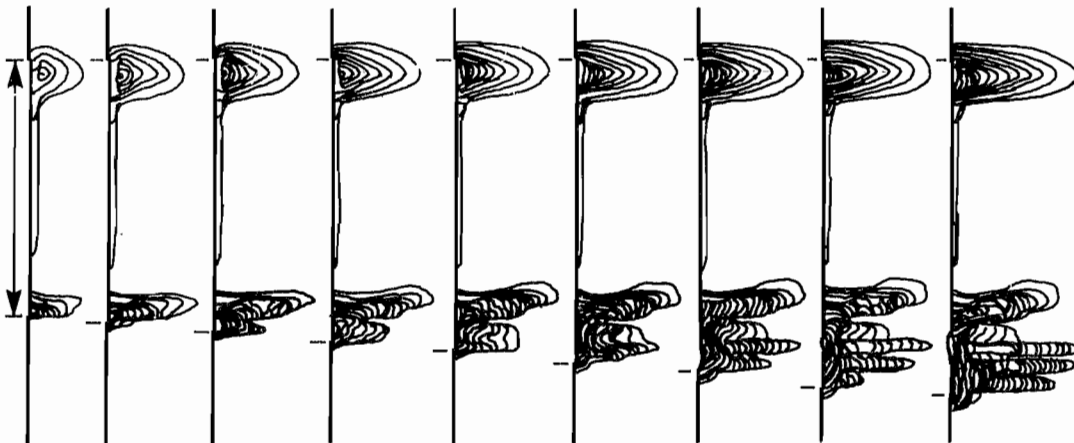
At present, there are no means of accurately determining the energy release rate within the model to determine conditions of plastic flow. It is not possible, therefore, to identify whether shear band formation relates to natural fracturing ahead of the stope face or to shear fracturing associated with rock bursts. Energy release rate estimates for the underhand longwall method are substantially lower than for the overhand method, but are still significant.



### LEGEND

↔ Mining extent

Figure 11.—Progressive development of shear bands around overhand cut-and-fill stope. Contours of plastic strain show localization of strain into shear bands as pillar was mined. Note dramatic extension of shear band at stope bottom when pillar height was reduced to 20 ft in the last step.



### LEGEND

↔ Mining extent

Figure 12.—Progressive development of shear bands around underhand cut-and-fill stope. Contours of plastic strain show development of shear band at mining face with each excavation step.

## DISCUSSION

On the basis of numerical modeling, some tentative estimates of the relative rock burst hazard for underhand longwall mining can be developed. It appears that underhand longwall mining will not eliminate shear fractures, and thus rock bursts, or at least seismic events, are likely to continue even though no sill pillar is created. Rock bursts could occur ahead of the face with some regularity as a result of shear-fracture formation or slip along existing fracture surfaces. Nonetheless, estimates based on stope closure and fracture length indicate that the magnitude of these events is likely to be smaller than when the overhand method is used. This conclusion is in line with smaller peak rates of energy release where the underhand longwall mining option is being used. Thus, the trade-off appears to be that there will be a greater number of seismic events, but these will have smaller magnitudes.

This conclusion is limited to an "average" seismic event by the complex geology typical of the Coeur d'Alene Mining District. It does not imply that large rock bursts will be eliminated, even those caused by shear rupture. Further analysis of the vulnerability of each stoping method to slip and shear-fracture rock bursts along discontinuities or planes of weakness, respectively, at unfavorable orientations to the stope may be able to give some estimate of the magnitude of "worst case" rock bursts. The analysis presented here has related strain localization to potential shear fracturing or, perhaps, slip along the surfaces of existing fractures. The energy associated with shear propagation in a given mining step was not determined, and it was therefore difficult to estimate if localized shear propagation was associated with normal fracturing around a stope or with damaging rock bursts.

## CONCLUSIONS

This exploration of a strain-softening constitutive law in the FLAC program found significant promise for modeling the shear fracture aspect of rock behavior. The ability to simulate shear-fracture failure in laboratory specimens and analyze mining methods for shear-fracture-related rock bursts was demonstrated. Still, there are serious problems in the current use of strain softening in numerical models. Some problems involve computational logic, but others reflect a poor understanding of the mechanics of rock behavior. Indeed, despite the general use of the classic

Mohr-Coulomb failure criterion throughout the rock mechanics field, there is a lack of fundamental knowledge of the mechanics of shear fracture. Thus, the quantitative results of these analyses cannot be trusted. However, the model showed good qualitative agreement between fairly crude mathematical models of strain softening and physical observations from laboratories and mines. As a fundamental understanding of shear fracturing improves, there is considerable potential for improving the ability to model shear-fracture-related rock bursts.

## REFERENCES

1. Adams, G. R., A. J. Jaeger, and C. Roering. Investigations of Rock Fracture Around Deep Level Gold Mine Stopes. Paper in Proceedings of 22d U.S. Symposium on Rock Mechanics: Rock Mechanics from Research to Application. MIT, Cambridge, MA, 1981, pp. 213-218.
2. Bieniawski, Z. T. Mechanism of Brittle Fracture of Rock. Part I. Theory of the Fracture Process; Part II. Experimental Studies; Part III. Fracture in Tension and Under Long-Term Loading. *Int. J. Rock Mech. and Min. Sci.*, v. 4, 1967, pp. 395-430.
3. Board, M. P., and M. D. Voegelé. Examination and Demonstration of Undercut and Fill Stopping for Ground Control in Deep Vein Mining. Paper in Application of Rock Mechanics to Cut and Fill Mining, ed. by O. Stephansson and M. J. Jones. (Proc. Conf. Lulea, Sweden, June 1-3, 1981). *Inst. Min. and Metall.*, London, England, 1981, pp. 331-353.
4. Crouch, S. L. Influence of Failed Rock on the Mechanical Behavior of Underground Excavations. Ph.D. Thesis, Univ. MN, Minneapolis, MN, 1971, 349 pp.
5. Cundall, P. A. Numerical Experiments on Localization in Granular Assemblies. Pres. at Workshop on Limit Analysis of Bifurcation Theory (Karlsruhe, W. Germany, Feb. 1988). *Ingenieur-Archiv* No. 59, 1989, pp. 148-159.
6. Cundall, P. A., and M. Board. A Microcomputer Program for Modeling Large-Strain Plasticity Problems. Paper in Numerical Methods in Geomechanics: Proceedings of the 6th International Conference on Numerical Methods in Geomechanics (Innsbruck, Austria, Apr. 1988). A. A. Balkema, Rotterdam, Netherlands, 1988, pp. 2101-2108.
7. DeBorst, R. Bifurcations in Finite Element Models With a Non-Associated Flow Law. *Int. J. Num. Meth. Geomech.*, v. 12, 1988, pp. 99-116.
8. \_\_\_\_\_. Nonlinear Analysis of Frictional Materials. Ph.D. Dissertation, Delft Univ. Technology, Delft, Holland, 1986, 230 pp.
9. Hedley, D. G. F. Rock Burst Mechanics. *CANMET Rep. MRL 87-118 (TR)*, 1988, 30 pp.
10. Hudson, J. A., E. T. Brown, and C. Fairhurst. Shape of the Complete Stress-Strain Curve for Rock. Paper in Stability of Rock Slopes: Proceedings, Thirteenth Symposium on Rock Mechanics, ed. by E. J. Cording (Univ. IL, Urbana, IL, 1971). *Am. Soc. Civ. Eng.*, 1972, pp. 773-796.
11. Iannacchione, A. T. Numerical Simulation of Coal Pillar Loading with the Aid of a Strain-softening Finite Difference Model. Paper in Rock Mechanics as a Guide for Efficient Utilization of Natural Resources: Proceedings of the 30th U.S. Symposium, ed. by A. W. Khair (WV Univ., Morgantown, WV, June 19-22, 1989). A. A. Balkema, Rotterdam, Netherlands, 1989, pp. 775-782.
12. Itasca Consulting Group, Inc. *FLAC Version 2.02*. Minneapolis, MN, 1987, 278 pp.
13. Jenkins, F. M., and R. R. Dorman. Ch. in Underhand Cut-and-Fill Stopping for Rock Burst Control. *BuMines IC 8973*, 1983, pp. 49-63.
14. Jenkins, F. M., T. J. Williams, and C. J. Wideman. Rock Burst Mechanism Studies at the Lucky Friday Mine. Pres. at 31st U.S. Symp. on Rock Mechanics, WV Univ., Morgantown, WV, June 18-20, 1990, 8 pp; available from Spokane Res. Cent., Spokane, WA.
15. Karwowski, W. J., W. C. McLaughlin, and W. Blake. Rock Preconditioning To Prevent Rock Bursts—Report on a Field Demonstration. *BuMines RI 8381*, 1979, 47 pp.
16. Lorig, L., and P. A. Cundall. Modeling of Reinforced Concrete Using the Distinct-Element Method. Ch. in *Fracture of Concrete and Rock*, ed. by S. P. Shah and S. E. Swartz. *Soc. Eng. Mech.*, Bethel, CT, 1987, pp. 459-471.
17. McMahon, T. Rock Burst Research and the Coeur d'Alene District. *BuMines IC 9186*, 1988, 49 pp.
18. McWilliams, P., M. McDonald, and M. Jenkins. Statistical Analysis of a Microseismic Field Data Set. Paper in Preprints, 2d Int. Symp. on Rockbursts and Seismicity in Mines (Univ. MN, Minneapolis, MN, June 8-10, 1988). 1988, pp. 235-246; available from P. McWilliams, Spokane Res. Cent., Spokane, WA.
19. Ortlepp, W. D. The Mechanism and Control of Rock Bursts. Ch. in *Rock Mechanics in Mining Practice*. *S. Afr. Inst. Min. and Metall.*, Johannesburg, S. Afr., 1983, pp. 257-281.
20. \_\_\_\_\_. The Mechanism of a Rock Burst. Paper in Proceedings of 19th Symposium on Rock Mechanics. Univ. NV, Reno, NV, 1978, pp. 476-483.
21. Parisseau, W. G., E. L. Corp, and M. E. Poad. Recent Experience in Calibration of Finite-Element Models by Back Analysis of Underground Hard Rock Mine Data. Paper in Proceedings of the International Symposium on Large-Scale Underground Mining (Lulea, Sweden, Nov. 6-7, 1985). *Centek*, Lulea, Sweden, 1985, pp. 175-183.
22. Ryder, J. A. Excess Shear Stress (ESS): An Engineering Criterion for Assessing Unstable Slip and Associated Rock Burst Hazards. Paper in Proceedings of Sixth International Conference on Rock Mechanics. A. A. Balkema, Rotterdam, Netherlands, v. 2, 1987, pp. 1211-1215.
23. Salamon, M. D. G. Rock Burst Hazard and the Fight for Its Alleviation in South African Gold Mines. Ch. in *Rock Bursts: Prediction and Control*. *Inst. Min. and Metall.*, London, England, 1983, pp. 11-36.
24. Spottiswoode, S. M. Source Mechanisms of Mine Tremors at Blyvooruitzicht Gold Mine. Paper in *Rock Bursts and Seismicity in Mines*, ed. by N. C. Gay and E. H. Wainwright (Proc., Symp. on Seismicity in Mines, Johannesburg, S. Afr.). *S. Afr. Inst. Min. and Metall.*, Johannesburg, S. Afr., Symp. Ser. 6, 1984, pp. 29-38.
25. Tanton, J. H., T. F. McCarthy, and T. O. Hagan. The Introduction of Stabilizing Pillars to Reduce Rock Bursts at Wester Deep Levels, Ltd. Paper in *Rock Bursts and Seismicity in Mines*, ed. by N. C. Gay and E. H. Wainwright (Proc., Symp. on Seismicity in Mines, Johannesburg, S. Afr.). *S. Afr. Inst. Min. and Metall.*, Johannesburg, S. Afr., Symp. Ser. 6, 1984, pp. 245-252.
26. Wagner, H. Determination of the Complete Load-Deformation Characteristics of Coal Pillars. Paper in *Advances in Rock Mechanics: Proceedings of the Third Congress of the International Society for Rock Mechanics* (Denver, CO, 1974). *Nat. Acad. Sci.*, Washington, DC, v. 2, pt. 3, 1974, pp. 1076-1081.
27. Whyatt, J. K., and M. J. Beus. In Situ Characterization of Overstressed Rock Fracturing. Paper in Proceedings of the 28th U.S. Symposium on Rock Mechanics. A. A. Balkema, Rotterdam, Netherlands, 1987, pp. 873-881.
28. Wilson, A. H. An Hypothesis Concerning Pillar Stability. *Min. Eng. (London)*, v. 131, No. 141, 1972, pp. 409-417.